Porro, cum Angulum sic, ut dictum est, definiverat, p. 67; subjungit, p. 68. *Quodsi magnitudines illæ sint duæ lineæ, comprehensus ab iis angulus, Planus vocabitur:* quasi quidem de *Triangulis sphæricis* nil unquam inaudiverit; nec alius esse possit superficialis angulus, quam in *Plano.*

Adhæc, illud *duarum pluriumve,* de *Lineis* non tuto dicitur. *Trium* enim linearum concursus, non angulum, sed angulos saltem duos, constituunt; non enim lineæ plures duabus ad unum superficialem angulum constituendum concurrunt. Item, cum p. 67. Angulum in genere per *duarum pluriumve,* &c. definiverat; Angulum p. 68. *una vel pluribus* superficiebus comprehensum ait (& unâ quidem angulum verticalem Coni comprehensum;) quasi quidem *una,* fuerit, *duæ* vel *plures.*

Insuper, quid demum illud est, quod per *brevissimam distantiam* insinuatum vult? Quippe in ipso concursus puncto, *Nulla est distantia*; extra illud, *nulla minima*: nulla utique assignari poterit, qua non sit minor: sed revera tota hæc, quam de *Angulo* notionem concipit, est parum sana. Definiendus utique est non per *distantiam* seu *remotionem,* sed per *Inclinationem.* qnod ex *Euclidis* definitione didicisset.

Deniq; (ne multus nunc sim) p. 171. in duabus his Quadraticarum æquationum formulis $aa-ca+dd=0$, & $aa+ca+dd=0$; utramque radicem *affirmativam* esse pronunciat. quod omnino secus est. Et quidem in priore, Radix utraque Affirmativa; sed in posteriore, Negativa utraque.

Atque hæc quadem, ex multis pauca, si non sufficiant, ut ex ungue Leonem æstimes, plura facile congerentur. Num autem hos *Incuria*, an *Inscitia*, errores fuderit (prout ipse *pag. ult.* distinguit) non determino. Vale.

*Hæc* Dn. *Wallisius* epistola una; cui postea submisit alteram, 18. *Julii* ad me scriptam, quam istoc mense, ob alia, non licebat typis committere; nec quidem licet hoc ipso: ne scil. hasce schedulas, publicationi variorum, idque imprimis sermone *Anglico,* destinatas, disceptationibus *Latinis* compleamus. Proxima occasione, quæ idem *Author* porro notanda invenit vel in *unico primo* Capite *Synopseos Laurentianæ,* Lectori (cum particularia flagitet Dn. Du Laurens) ob oculos sistemus.

## *An Account of Two Books.*

I. *R. de GRAAF Med. D. de VIRORUM ORGANIS GENERATIONI INSERVIENTIBUS, &c.* Ludg. Bat. 1668. *in* 12°.

This Treatise was promised by the Author in a printed *Epistle* of his, which we gave an account of in *April* last, *Num.* 34. p. 663. There being at the same time publisht a *Prodromus* of *Joh. Van Horne,* suspecting, that the Observations of *De Graef* were much the same with his upon this Subject; we do now upon the perusal of this Book, find chiefly these considerable Differences between them.

p. 663.

*First*, the ſaid *Van Horne* makes the *Spermatick Artery* in man to goe to the *Teſticles* in a *winding*, but *De Graaf*, in a *ſtreight* way.

*Secondly*, the *former* affirms, that the *vaſa deferentia* have no communication with the *veſiculæ ſeminales*; but the latter maintains, and demonſtrateth it to the Ey, there is ſo great a commerce betwixt them, *ut ſemen dum à Teſtibus per vaſa differentia affluens in Urethram effluere nequit, propter carunculam clauſam; neceſſariò influat in Veſiculas, in iiſque pro futuro coitu reſervetur.*

*Thirdly*, the *former* is of opinion, *triplicem eſſe materiam ſeminis*; but *De Graaf* will have but *one only*; anſwering the Arguments, uſed both by *Van Horne* and *Dr. Wharton* to prove that *triplicity*.

But that, which *De Graaf* much inſiſts on in this Book, is, to ſhew what is the *true Subſtance* of the *Teſticles*, and to vindicate the Diſcovery thereof to himſelf; affirming poſitively, that no man, before him ever knew the truth of it. * For the making out of which, he firſt denyeth, that the *Teſtes* are *glandulous*, or *pultaceous*; and then affirms that their ſubſtance is nothing elſe but a *Congeries minutiſſimorum vaſculorum ſemen conficientium, quæ ſi abſque ruptione diſſoluta ſibi invicem adnecterentur, facile viginti ulnarum longitudinem excederent.* Which he affirms, he can prove by ocular Demonſtration.

* See the Letter of Doctor *Tim. Clark*, N. 35. p. 681.

Then he ſheweth, how the ſeminal veſſels paſs *è Teſtibus ad Epididymides*, vid. not by *one* Trunck (as Dr. *Highmore* thinks) but by 6. or 7. ſmall *ductus*'s; aſſigning the cauſe, why Doctor *Highmore* did not ſee them.

Further he examines, An ſemen in teſtibus conficiatur; utrum ex Sanguine vel ex Lympha? quomodo elaboretur, craſſeſcat, lacteſcat: qua via à Teſtibus ad Urethram excurrat.

Moreover he endeavours to prove, *Veſiculas ſeminales ordinatas eſſe non ſeminis generationi, ſed receptieni & aſſervationi.*

He alſo obſerveth concerning the *ſeminal matter*, that 'tis compoſed *ex duplici materia*, which after *Ariſtotle*, he calls λόγον σπερματικὸν καὶ ὄγκον σπερματικόν, conſidering this twofold matter like *Dough* and *Ferment*, this infecting and quickning that, and the groſſer part being a conſervatory and vehicle to that, which is moſt elaborate.

When he examins the *Penis*, he taketh notice, *Omnes hactenus*

*nus Anatomicos perperam assignasse usum musculorum Penis*, *quos* Erectores *appellant*; *Eorum quippe provinciam non esse, Penem erigere, & dilatare Urethram, cum omnis Musculi actio sit contractio, quæ extensioni contraria est*; *eos potius* Penem *versus interiora retrahere quam erigere: Interim, hosce Penis Musculos, coarctando corpora nervosa circa eorum exortum*, *materiam seminalem versus Penis partem anteriorem propellere, atque hac ratione corporum nervosorum distensione erectionem augere.*

Before we conclude this Account, we cannot but take notice, that the Author occasionally inserts in this Book divers curious and remarkable Examples and Observations; some whereof are.

1. Concerning those, that are born, either *absque Testibus*; or, *cum Testiculo uno*; or, *cum tribus*, *idque hæreditario per aliquot familias, admodum fæcundas.*

2. About the *situs præternaturalis Testiculorum, generationis tamen virtutem non impedientis.*

3. Concerning *lactescent Bloud* in a man living at *Delft* in *Holland*, whose Bloud alwayes turn'd into Milk, when let out either by venæ-sections, or by bleeding at the Nose, or by a wound. V. pag. 84, 85. Compare *Numb. 6.* pag. 100, 117, 118. and *Numb.* 8. pag. 139. of these *Transactions*.

4. Concerning the strange alteration made in Femals, *ab Aura seminali*: quod confirmat exemplo felis, diu sugentis (idque ad integram fere sui nutritionem) lac mammarum caniculæ, per aliquot annos à coitu prohibitæ; deinceps vero, postquam catella admiserat canem, nunquam ab eo tempore lac ex mammis ejus exsugere volentis.

5. About a strange *Hæmorrhagy* per *Penem*, which amounted to 14. pound, in a Porter of 52. years old, falling down with a heavy load upon a board, laid over a ditch, and so turning about, when the said porter trod upon it, that it cast him down upon its edge, turn'd between his legs; yet the Patient by the skill and care of our Author recover'd.

6. Various Observations of Clysters and Suppositories, cast up by Vomits, p. 195, 196.

7. Several wayes of performing unbloudy dissections of Animals, p. 228, 229, &c.

*II. LO-*

## II. *LOGARITHMOTECHNIA NICOLAI MERCATORIS. Concerning which we ſhall here deliver the account of the Judicious Dr.* I. Wallis, *given in a Letter to the Lord Viſ-count* Brouncker, *as follows* ;

Incidebam heri (Illuſtriſſime Domine) in D. *Mercatoris Logarithmotechniam*, nuper editam. Quæ ita mihi placuit, ut non prius dimiſerim quàm perlegiſſem totam. Et quamquam pauca quædam, Phraſeologiam quod ſpectat ſeu loquendi formulas nonnullas, mutata mallem; ſunt tamen ipſa ſenſu ſuo ſana: Eaque quæ ſuperſtruitur Doctrina, Logarithmos expedite atque ſubtiliter conſtruendi, perſpicue ſatis atque ingenioſe traditur.

Quæ huic ſubjungitur *Quadratura Hyperbolæ*, elegans admodum eſt atque ingenioſa. Nempe ad hunc ſenſum. V.Fig.1.

Poſtquam in Hyperbola MBF, (cujus Aſymptotæ AN, AH, ad angulum rectum coeunt) oſtenderat, prop. 14, Rectangula BIA, FHA, spA, &c. (ductis BI, FH, sp, &c, parallelis Aſymptotæ AN,) invicem eſſe æqualia; adeoque latera habere reciproce proportionalia; (quæ nota eſt Hyperbolæ proprietas:) Poſitis $AI = BI = 1$, & $HI = a$: oſtendit, prop. 15. $FH = \frac{1}{1+a}$ (Nempe propter analogiam AH. AI :: BI. FH. hoc eſt. $1+a.\ 1 :: 1.\ \frac{1}{1+a}$. Sed & (quod Dividendo 1, per $1+a$ oſtenditur,) $\frac{1}{1+a} = 1 - a + a^2 - a^3 + a^4$ &c. ] (continuatis deinceps, ipſius $a$ poteſtatibus, alternatim negatis & affirmatis.)

$$
\begin{array}{llllll}
1+a)\,1\,(\,1, -aa^2, +, a^3, +, a^4, \&c \\
\underline{1+a} \\
\quad -a \\
\underline{\quad -a & -a^2} \\
 & +a^2 \\
 & \underline{+a^2 & +a^3} \\
 & & -a^3 \\
 & & \underline{-a^3 & a^4} \\
 & & & +a^4 \\
 & & & \&c.
\end{array}
$$

Cumque hoc perinde obtineat, ubicunque ultra punctum I, ponatur H. Poſitis, ut prius $AI = 1$; hujuſque continuatione qualibet, ut $Ir = A$; quæ intelligatur in æquales partes innumeras dividi, quarum quælibet, ut Ip. pq, &c. dicatur $a$; adeoque Ip, Iq, &c, ſint $a$, $2a$, $3a$, &c. uſque ad A: Quæ his reſpondent rectæ ps, qt, &c. uſque ad *ru*, (ſpatium BI ru complentes) ſunt,

1—a

$$1 - a + a^2 - a^3 + a^4 \text{ \&c.}$$
$$1 - 2a + 4a^2 - 8a^3 + 16a^4 \text{ \&c.}$$
$$1 - 3a + 9a^2 - 27a^3 + 81a^4 \text{ \&c.}$$

& sic deinceps usque ad

$$1 - A + A^2 - A^3 + A^4 \text{ \&c.}$$

Cum itaque sint $1 + 1 + 1$ &c. (usque ad ultimum) $= A$
$a + 2a + 3a$ &c. (usque ad $A$) $= \frac{1}{2}A^2$
$a^2 + 4a^2 + 9a^2$ &c. (usque ad $A^2$) $= \frac{1}{3}A^3$
$a^3 + 8a^3 + 27a^3$, &c. (usque ad $A^3$) $= \frac{1}{4}A^4$
& sic deinceps:

(quod ostendit ille prop. 16, estque à me alibi demonstratum:) Recte colligit, prop. 17. Expositum spatium Hyperbolicum BIru $= A - \frac{1}{2}A^2 + \frac{1}{3}A^3 - \frac{1}{4}A^4 + \frac{1}{5}A^5$, &c. Adeoque si (assignato, ipsi $A = Ir$, valore suo in numeris, ut res postulaverit,) distribuantur in duas classes $A, \frac{1}{3}A^3, \frac{1}{5}A^5$, &c. (potestates affirmatæ,) & $\frac{1}{2}A^2, \frac{1}{4}A^4$, &c. (potestates negatæ;) harumque Aggregatum, ex Aggregato illarum, subducatur; Residuum erit ipsum BIru spatium Hyperbolicum.

Nequis autem operam lusum iri existimet,, propter Addendorum seriem in utraque classe infinitam; adeoque non absolvendam: Hinc incommodo medelam (tacitus) adhibet: ponendo $A = 0{,}1$, vel $A = 0{,}21$, aliive fractioni decimali æqualem, adeoque minorem quam 1: (Hoc est, sumpta Ir minore quam $AI = 1$.) Quo fit, ut posteriores ipsius A potestates tot gradibus infra Integrorum sedem descendant, ut merito negligi possint.

Exempli gratia; positis $AI = 1$, & $Ir = 0{,}21$. erit

$A = 0{,}21$

| | |
|---|---|
| $\frac{1}{3}A^3 = 0{,}003087$ | $\frac{1}{2}A^2 = 0{,}02205$ |
| $\frac{1}{5}A^5 = 0{,}000081682$ | $\frac{1}{4}A^4 = 0{,}000486202$ |
| $\frac{1}{7}A^7 = 0{,}000002572$ | $\frac{1}{6}A^6 = 0{,}000014294$ |
| $\frac{1}{9}A^9 = 0{,}000000088$ | $\frac{1}{8}A^8 = 0{,}000000472$ |
| $\frac{1}{11}A^{11} = 0{,}000000003$ | $\frac{1}{10}A^{10} = 0{,}000000016$ |

$+ 0{,}213171345 - 0{,}022550984 = 0{,}190620361 = BIru$

Quæ est brevis Synopsis Quadraturæ suæ satis elegans.

Dissimulandum interim non est; siquis totius BIHF spatii (cujus latus IH longius intelligatur quam AI) quadraturam postulet; rem non ita feliciter successuram: propter medelam, quam modo diximus, malo minus sufficientem. Cum enim jam ponenda sit $A > 1$; manifestum est, posteriores ipsius potestates, altius in Integrorum sedes penetraturas, adeoque minime negligendas.

Huic autem incommodo, levi constructionis immutatione, facile subvenitur. Vid. Fig. 1.

Cæteris utique ut prius constructis; Quadrandum exponatur HFur spatium

tium; (cujuscunque fuerit longitudinis A H; puta major minorve quam A I, vel huic æqualis: sumptoque ubivis inter A & H, puncto r; puta ultra citrave punctum I, vel in ipso I puncto:) Ponantur autem (non, ut prius $AI = 1$, & $Ir = A$: sed) $AH = 1$; & $Hr = A$, quæ intelligatur in æquales partes innumeras dividi, quarum quælibet sit *a*. Erunt itaque, post $AH = 1$, reliquæ deinceps decrescentes $1-a$, $1-2a$, $1-3a$, &c. usque ad $Ar = 1 - A$. Item, propter æqualia Rectangula FHA, urA, BIA, &c. puta, $= b^2$: Erit $HF = \frac{b^2}{1}$; reliquæque deinceps $\frac{b^2}{1-a}$, $\frac{b^2}{1-2a}$, $\frac{b^2}{1-3a}$, &c, usque ad $ru = \frac{b^2}{1-A}$ spatium HFur complentes. (Quæ omnia ostensa sunt, in mea *Arithmetica Infinitorum*, prop. 88, 94, 95.)

Factaque Divisione; reperietur $\frac{b^2}{1-a} = b^2 + b^2a + b^2a^2 + b^2a^3 + b^2a^4$, &c. Hoc est, $b^2$ in $1 + a + a^3 + a^2 + a^4$, &c. (sumptis ipsius *a* potestatibus continue sequentibus affirmatis omnibus.) Cumque de reliquis idem sit judicium; erunt rectæ omnes, ipsis HF & ru interjectæ,

$$
\begin{array}{l}
1-a)\,b^2\,(b^2, + b^2a, + b^2a^2, + b^2a \text{ \&c.}\\
\underline{b^2 - b^2a}\\
\quad + b^2a\\
\quad \underline{+ b^2a - b^2a^2}\\
\qquad + b^2a^2\\
\qquad \underline{+ b^2a^2 - b^2a^3}\\
\qquad\quad + b^2a^3\\
\qquad\quad \underline{+ b^2a^3 - b^2a^4}\\
\qquad\qquad + b^2a^4\\
\qquad\qquad \text{\&c.}
\end{array}
$$

$$
\left.\begin{array}{l}
1 + a + a^2 + a^3 + a^4 \text{ \&c.}\\
1 + 2a + 4a^2 + 8a^3 + 16a^4 \text{ \&c.}\\
1 + 3a + 9a^2 + 27a^3 + 81a^4 \text{ \&c.}\\
\text{\& sic deinceps usque ad}\\
1 + A + A^2 + A^3 + A^4 \text{ \&c.}
\end{array}\right\} \text{in } b^2.
$$

Omniumq; Aggregatū, $A + \frac{1}{2}A^2 + \frac{1}{3}A^3 + \frac{1}{4}A^4 + \frac{1}{5}A^5$ &c, in $b^2 = FHru$.
(per *Arithm. Infin.* prop. 64.)

Exempli gratia.
Positis $AH = 1$.
$Hr = A = 0,21$
$AI = b = 0,\ 1$
Adeoque $b^2 = 0,01$

Erunt $A = 0,21$
$\frac{1}{2}A^2 = 0,02205$
$\frac{1}{3}A^3 = 0,003087$
$\frac{1}{4}A^4 = 0,00048623 -$
$\frac{1}{5}A^5 = 0,000081682 +$
$\frac{1}{6}A^6 = 0,000014294 +$
$\frac{1}{7}A^7 = 0,000002573 -$
$\frac{1}{8}A^8 = 0,000000473 -$
$\frac{1}{9}A = 0,000000088 +$
$\frac{1}{10}A^{10} = 0,0000000017 -$
$\frac{1}{11}A^{11} = 0,000000003 +$

Horum summa — 0,235722333
Ducta in $b^2 = 0,01$
Exhibet ——— 0,00235722333 = FHru

Qua-

Qualium 1. = A N G N $\left\{\begin{array}{l}\text{Quadrato,}\\ \text{Rhombo,}\end{array}\right\}$ ſi angulus A ſit $\left\{\begin{array}{l}\text{Rectus.}\\ \text{Obliquus.}\end{array}\right.$

Quæ quidem tam abſoluta eſt tamque expedita Hyperbolæ quadratura, ut neſciam an melior ſperari debeat.

Atque hæc ſunt quæ hac de re ſcribenda duxi. Quæ ſi D. *Mercatori* impertiveris; non diſplicebit, credo, hæc ſuæ Quadraturæ facta acceſſio.

Poſſe hæc ad *Logarithmorum* inventionem accommodari, non eſt quod moneam: Sed & ad *Summam Logarithmorum* inveniendam: (quam inquirit ille prop. 19.) Nempe, Poſitis A H = 1, A I = I B = b, (ut prius) planoque B I H F = pl. Erit pl — $b^2$ + $b^3$ = B I p s + B I q t + B I r u, &c. uſque ad B I H F. Si autem non ab ipſa B I incipiatur; ſed ultra citrave, puta à *p s*: Poſita p H = a & p s F H = p l. erit ( univerſaliter ) p s t q + p s u r &c (uſque ad p s F H) = p l — $ab^2$: qualium 1, æquetur cubo ipſius A H.) Quod alias, ſi opus erit, demonſtrabitur. Tu interim, Illuſtriſſime Domine, Vale.

*Oxon. d.* 8. *Julii*, 1668.

Fig 1.

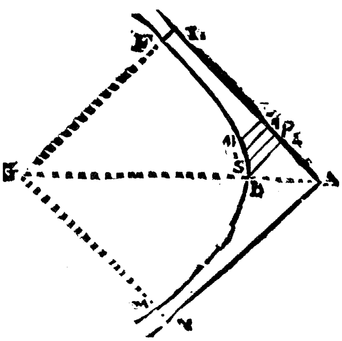
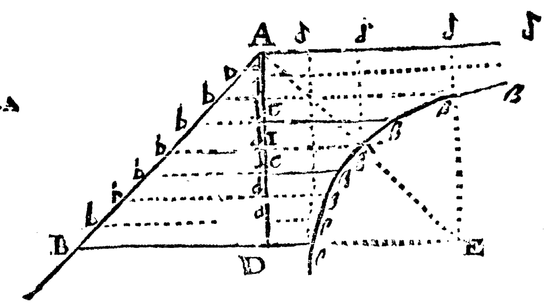


Fig 2



## The Demonſtration

Promiſed at the end of the foregoing Letter, follows in another from the ſame Author to the ſame Noble Lord, thus;

Petis (Illuſtriſſime Domine) per literas tuas *Aug*.3. datas (quas heſterna nocte accepi) ut demonſtrare velim methodum meam, Logarithmorum ſummam inveniendi, quam literis meis *Julii* 8. datis, breviſſime inſinuaveram.

Quæ quidem, cum ſit cum Ungularum doctrina (quam alibi trado) connexa; opus erit ut utramque ſimul exponam: ſed & rem totam (quam D. *Mercatoris*

*ventoris* figuræ & methodo quantum res ferebat accommodaveram) ad principia mea revocatam ab origine repetam. V. Fig. 2.

Ostensum est, in mea *Arithmetica Infinitorum*, prop. 95. Spatium Hyperbolicum ADββδ (in infinitum continuatum à parte βδ, sed à parte Dβ ubivis terminatum,) Figuram esse quam ex *Primanorum Reciprocis* conflatam appello, Prop. 88. definitam: Cujus nempe Ordinatim—applicatæ dβ, dβ, sint Primanis (seu Arithmetice proportionalibus) db, db, (Triangulum complentibus) adeoque ipsis dA, dA, (suis à vertice distantiis) Reciproce Proportionales. Hoc est, (posito AD $=$ d; & rectangulo ADβ $= b^2$; particulisque infinite exiguis a, a, &c;) si à vertice Aδ incipias $\frac{b^2}{o}$, $\frac{b^2}{a}$, $\frac{b^2}{2a}$, $\frac{b^2}{3a}$, &c. usque ad $\frac{b^2}{d} =$ Dβ: vel, si à base Dβ incipias, $\frac{b^2}{d}$, $\frac{b^2}{d-a}$, $\frac{b^2}{d-2a}$, $\frac{b^2}{d-3a}$, &c. usque ad $\frac{b^2}{d-d} =$ Aδ infinitæ, (nempe, si ad Verticem usque processum continuaveris;) vel, usque ad $\frac{b^2}{d-A} =$ Cβ, (posito DC $=$ A,) si continuaveris usque ad Cβ, ubivis intra Aδ & Dβ sumptam. (Adeoque omnium Aggregatum, $\frac{b^2}{d} + \frac{b^2}{d-a} + \frac{b^2}{d-2a} + \frac{b^2}{d-3a}$, &c, est ipsum DCββ planum.)

Manifestum itaque est, (& ibidem prop. 94. ostensum) si intelligantur singulæ dβ, in suas à vertice distantias *Ad*, ductæ; hoc est, $\frac{B^2}{a}$ in *a*, $\frac{B^2}{2a}$ in 2a, (& sic de reliquis;) erunt omnia rectangula A dβ; hoc est, rectarum dβ momenta respectu Aδ, (intellige, facta ex magnitudine in distantiam ductâ;) seu plana semiquadrantalem Ungulam (cujus acies Aδ) complentia, (eisdem dβ rectis perpendiculariter insistentia;) invicem æqualia. Quippe singula $= b^2$. (Quorum cum unum sit AIVδ quadratum, erit AI $=$ b.)

Adeoque Totius ADββδ (plani infiniti) seu omnium dβ illud complentium, momentum respectu rectæ Aδ, (ut axis æquilibrii;) seu Ungula semiquadrantalis eidem ADββδ insistens (aciem habens Aδ;) sunt totidem $b^2$; hoc est, d $b^2$. (Ungula magnitudinis finitæ plano infinitæ magnitudinis insistens.) Ejusque pars plano ACββδ insistens (propter AC $=$ d — A.) similiter ostendetur æqualis ipsi d — A in $b^2$. ductæ; hoc est, d $b^2$ — A $b^2$. Adeoque pars reliqua, ipsi DCββ insistens, æqualis ipsi A $b^2$. Quod itaque est ejusdem DCββ momentum respectu Aδ.

Atque hoc momentum per plani $DC\beta\beta$ magnitudinem, puta per $pl$, divisum; exhibet plani distantiam Centri gravitatis ab $A\delta$, $\frac{ab^2}{pl}$: adeoque distantiam ejusdem a $D\beta$, $d - \frac{ab^2}{pl}$.

Hæc itaque à $D\beta$ distantia, in $pl$ (plani magnitudinem) ducta; exhibet $dpl - Ab^2$ ejusdem $DC\beta\beta$ momentum respectu $D\beta$; seu Ungulam eidem $DC\beta\beta$ insistentem, cujus acies sit $D\beta$.

Hæc denique Ungula (cujus altitudo, in $D\beta$, nulla sit, sed, in $C\beta$, ipsi $DC$ æqualis:) si ex planis ipsi $DC\beta\beta$ parallelis conflari intelligatur; erunt ea, $CD\beta\beta$, $Cd\beta\beta$, & sic deinceps; hoc est, aggregatum omnium $Cd\beta\beta$, $Cd\beta\beta$, usque ad $CD\beta\beta$.

Sunt autem ea plura (ut ex *Gregorii de Sancto Vincentio*, aliorumque post illum, doctrina constat) tanquam Logarithmi Arithmeticè proportionalium $Cd$, $Cd$, &c. usque ad $CD$; (seu $a$, $2a$, $3a$, &c. usque ad $A$. Ergo Ungula ipsa, est eorundem Aggregatum. Hoc est (posito $D = 1$,) $dpl - Ab^2 = pl - Ab^2$. Quod ostendendum erat.

Porro; cum sit $\frac{b^2}{d-a}$ ($= d\beta$) $= \frac{b^2}{d} + \frac{ab^2}{d2} + \frac{a^2b^2}{d3} + \frac{a^3b^2}{d4}$ &c (Quod dividendo $b^2$ per $d-a$, patebit:) vel, posito $d = 1$, (quò ipsius $d$ potestates omnes deleantur,) $b^2 + ab^2 + a^2b^2 + a^3b^2$ &c. seu $1 + a + a^2 + a^3$, &c. in $b^2$. & similiter $\frac{b^2}{d-2a} = \frac{b^2}{d} + \frac{2ab^2}{d^2} + \frac{4a^2b^2}{d^3} + \frac{8a^3b^2}{d^4}$ &c. $= b^2 + 2ab^2 + 4a^2b^2 + 8a^3b^2$ &c. $= b^2$ in $1 + 2a + 4a^2 + 8a^3$, &c. & similiter in reliquis:

Erunt omnes $d\beta$, (spatium $DC\beta\beta$ complectentes,)

$$\left.\begin{matrix} 1 + a + a^2 + a^3 + a^4 \text{ \&c.} \\ 1 + 2a + 4a^2 + 8a^3 + 16a^4 \text{ \&c.} \\ 1 + 3a + 9a^2 + 27a^3 + 81a^4 \text{ \&c.} \\ \text{\& sic deinceps usque ad} \\ 1 + A + A^2 + A^3 + A^4 \text{ \&c.} \end{matrix}\right\} \text{in } b^2.$$

Adeoq; (per Arithm. Infin. prop. 64.) omnium Aggregatum, seu ipsum $DC\beta\beta$ spatium, erit ———

$$A + \tfrac{1}{2}A^2 + \tfrac{1}{3}A^3 + \tfrac{1}{4}A^4 + \tfrac{1}{5}A^5 \text{ \&c, in } b^2 = pl.$$

Qualium $1 = ABE\delta$ Quadrato vel Rhombo

Ideoque, Plani $DC\beta\beta$ momentum respectu $D\beta$; seu semiquadrantalis Ungula eidem insistens cujus acies sit $D\beta$; seu planorum aggregatum ipsam constituentium; seu Logarithmorum summa quos ea repræsentant, $dpl - Ab^2 = pl - Ab^2$, $= \frac{1}{2}A^2 + \frac{1}{3}A^3 + \frac{1}{4}A^4 + \frac{1}{5}A^5$ in $b^2$:

Qualium

Qualium Cubus (seu Rhombus solidus) A D E ♂ sit 1.

Si vero non ponatur $d=1$, sed cujuscunque magnitudinis: erit saltem

$$\frac{A}{d}+\frac{A^2}{2d^2}+\frac{A^3}{3d^3}+\frac{A^4}{4d^4} \text{ \&c. in } b^2. = pl.$$

Vel (posito $\frac{A}{d}=e$) erit $e+\frac{1}{2}e^2+\frac{1}{3}e^3+\frac{1}{4}e^4$ &c. in $b^2 = pl.$ Qualium $d^2 =$ A D E ♂ Quadrato vel Rhombo.

Ungulaque (ut prius) $d\,pl - A\,b^2$. Qualium $d^3 =$ A D E ♂ Cubo, vel (si angulus A sit obliquus) Rhombo solido.

Cumque A (posito $d=1$) vel *e* (quicunque ponatur valor ipsius *d*) sit minor quam 1, (propter $A < d$:) illius potestates posteriores ita continue decrescunt, ut tandem negligi possint; planique valor *pl.* exhibeatur quantumlibet vero propinquus.

Atque hæc est, Illustrissime Domine, Methodi, quam innuebam, ex meis principiis deductio, & demonstratio brevis. Vale. *Oxon.* d. 5. *Aug.* 1668.

## *Some Illustration* *Of the* Logarithmotechnia *of M.* Mercator, *who communicated it to the Publisher, as follows.*

Si quorum in manus incidit Logarithmotechnia mea, non inviti, opinor, adspicient paucula hæc exempla, miram istius methodi facilitatem cum summa præcisione conjunctam ostendentia.

| Exponentes | Unitatis ordo | | Binarii ordo | |
|---|---|---|---|---|
| 1 | 1 | i | 2 | 2 |
| 2 | 1 | 0,5 | 4 | 2 |
| 3 | 1 | 0,333333 | 8 | 2,666666 |
| 4 | 1 | 0,25 | 16 | 4 |
| 5 | 1 | 0,2 | 32 | 6,4 |
| 6 | 1 | 0,166666 | 64 | 10,666666 |
| 7 | 1 | 0,142857 | 128 | 18,285714 |
| 8 | 1 | 0,125 | 256 | 32 |
| 9 | 1 | 0,111111 | 512 | 56,888888 |
| 10 | 1 | 0,1 | 1024 | 102,4 |

Duo sunt ordines tabellæ, prior unitatis, alter binarii propago, quorum uterque denorum numerorum primorum Log-os producit, præter compositorum Log-os, qui & ipsi requiruntur.

 Ex

Ex primo ordine

1
05
0333333333
025
02
0166666
01428
0125
011
01

$+$ 10033534772
$-$ 502516792

$105360515\,64\frac{9}{10}$
$95310179\,80\frac{10}{11}$

1
.05
.0333333
025
02

$+$ 10000333353
$-$ 500025

$10050335853\frac{99}{100}$
$9950330853\frac{100}{101}$

Parimodo ex eodem ordine procedunt rationes $\frac{999}{1000}$, $\frac{1000}{1001}$, $\frac{9999}{10000}$, $\frac{10000}{10001}$, $\frac{99999}{100000}$, $\frac{100000}{100001}$.

Ex secundo ordine.

2
2
2666666666
4
64
1066666
182857
32
5689
1024
186
341
630

$+$ 20273255404
$-$ 2041099724

2
.2
.2666666
4
.64
10

$+$ 20002667306
$-$ 200040010

$20202707316\frac{98}{100}$
$19802627296\frac{100}{102}$

Haud secus ex eodem ordine eliciuntur rationes $\frac{998}{1000}$, $\frac{1000}{1002}$, $\frac{9998}{10000}$, $\frac{10000}{10002}$, $\frac{99998}{100000}$, $\frac{100000}{100002}$, &c.

| | | |
|---|---|---|
| 1 | | $22314355128\frac{8}{10}$ |
| 2 | | $18232155680\frac{10}{12}$ |
| 3 | 1 + 2 | $40546510808\frac{8}{12} = \frac{2}{3}$ |
| 4 | ex pc. pag. | $10536051564\frac{9}{10}$ |
| 5 | 2 + 4 | $28768207244\frac{9}{12} = \frac{3}{4}$ |
| 6 | 3 + 5 | $69314718052\frac{2}{4} = \frac{1}{2} = L\,2^{rii}$ |
| 7 | 6 × 3 | $207944154156\frac{1}{8} = L\,8^{rii}$ |
| 8 | 1 + 7 | $230258509284\frac{1}{10} = L\,10^{rii}$ |
| 9 | ex pc. pag. | $9531017980\frac{10}{11}$ |
| 10 | 8 + 9 | $239789527264\frac{1}{11} = L\,11^{rii}$ |
| 11 | 3 + 6 | $109861228860\frac{1}{2} + \frac{2}{3} = L\,3^{rii}$ |

Simi-

Similes ordines à $3^{rio}$, $4^{rio}$, & quovis alio numero derivari possunt, suas quisque rationes exhibiturus.

Acquisito Log-o $10^{rii}$, conficienda est statim tabella reducendorum Log-orum naturalium ad Tabulares, ut quævis ratio, simul ac inventa est, reducatur ad mensuram tabularium; ita enim Log-i compositorum, quorum ope ad primorum Log-os descenditur, simul fient Tabulares absque reductione.

Fiat igitur, ut Log-us $10^{rii}$ non-tabularis 2302585, ad tabularem 10000000; ita 1, ad 4,3429448. Hic numerus bis, ter, quater & pluries sumptus constituit tabellam reducendorum Log-orum naturalium ad tabulares, quam hic subjectam vides.

| | |
|---|---|
| 1 | 043429448190 |
| 2 | 086858896380 |
| 3 | 130288344570 |
| 4 | 173717792761 |
| 5 | 217147240951 |
| 6 | 260576689141 |
| 7 | 304006137332 |
| 8 | 347435585522 |
| 9 | 390865033712 |

Hujus igitur ope tabellæ, rationis $\frac{98}{100}$ mensura naturalis 20202707316 reducitur ad tabularem hoc modo:

| | |
|---|---|
| 2 | 086858896381 |
| 0 | 0 |
| 2 | 0868588964 |
| 0 | 0 |
| 2 | 08685890 |
| 7 | 3040061 |
| 0 | 0 |
| 7 | 30401 |
| 3 | 1303 |
| 1 | 043 |
| 6 | 26 |
| | 87739243069 |

Tum à Log-o $100^{rii}$ 20000000000000
auferatur rationis $\frac{98}{100}$ mensura 87739243069
restat 19912260756031 = L 98
unde ablato Log-2ii 3010299956640
restat ——— 16901960800291 = L 99
cujus semis ——— 8450980400145 = L 7

Item rationis $\frac{100}{102}$ mensura naturalis 19802627296 reducta, fit 86001717619.

Ergo jà Log-o $100^{rii}$ 20000000000000
adde rationis $\frac{100}{102}$ mensura 86001717619
fit ——— 20086001717619 = L 102
unde ablato Log-o $6^{rii}$ 7781512503836
restat ——— 12304489213783 = L 17

Hic tabula numerorum primorum egregium usum præstare potest.

Sed & ejusdem primi 17 Log-um absque ambage invenire datur, dicendo: 20. 17 :: 10. 8|5; tum differentiæ inter 10 & 8|5 (nimirum 1|5) sumendo quadrati semissem, cubi trientem, &c. tractandoque istum ordinem, ut suprà, inveniemus simul Log-os absolutorum 23, 197, 203, 1997, 2003, &c.

L

| | | |
|---|---|---|
| 1 | 1, 5 | 1, 5 |
| 2 | 2, 25 | 1, 125 |
| 3 | 3, 375 | 1, 125 |
| 4 | 5, 0625 | 1, 265625 |
| 5 | 7, 59375 | 1, 51875 |
| 6 | 11, 390625 | 1, 8984375 |

15
1125
1125
1265625
1518
189
22
+15114040
— 1137845
16251885 $\frac{17}{20}$
13976195 $\frac{20}{23}$

Cæterùm isthæc omnia, & longè plura ex prop. 13, 15, & 16 Logarithmotechniæ nostræ apertè derivantur, non magis considerando hyperbolam, quàm si ea nusquam in rerum natura extitisset. Quare frustra sunt, qui hyperbolam ad constructionem Logarithmorum vel hilum conferre autumant; imo Logarithmorum ope quadrare hyperbolam, verius est. Id quod exemplo ostendere haud pigebit. In diagrammate (Fig. 1.) sit AH 74305816 partium, qualium AI est 1, & oporteat invenire aream BIHF.

Opus est ad eam rem tabella subjecta, quæ continet Log-os naturales suprà acquisitos, in priori columna ab 1 usque ad 9, in altera à 10 usque ad 1000000000.

| | | |
|---|---|---|
| 1 | 00000000000 | 02, 30258509299 |
| 2 | 69314718052 | 04, 60517018599 |
| 3 | 109861228860 | 06, 90775527898 |
| 4 | 138629436104 | 09, 21034037198 |
| 5 | 160943791232 | 11, 51292546497 |
| 6 | 179175946912 | 13, 81551055796 |
| 7 | 194591014904 | 16, 11809565096 |
| 8 | 207944154156 | 18, 42068074395 |
| 9 | 219722457720 | 20, 72326583695 |

Tum prima figura numeri dati semper distinguatur à sequentibus separatrice, hoc modo: 7, 4305816, & ipsi primæ figuræ semper adjiciatur 1, ita conflantur, hoc loco, 8. Quærenda est nunc rationis 8 ad 7,4305816 mensura naturalis. Id ut fiat commodius, dic: ut 8 ad 7, 4305816; ita 1 ad 0, 9288227, hunc quartum proportionalem aufer ab 1, reliquum 0, 0711773 voco potestatem primam, quæ ducenda est in se ita, ut in facto idem numerus partium extet, qui erat in ipso 0, 0711773; productum 0, 0050662 est potestas secunda, quæ rursus ducatur in primam 0, 0711773, ut idem numerus partium exstet, prodit 0, 0003606, quæ est tertia potestas; & eodem modo invenitur quarta 0, 0000256, & quinta 0,0000018. Deinde

Potestas

| | | |
|---|---|---|
| Poteſtas prima | 0, 0711773 | addantur |
| Et ſecundæ ſemis | 25331 | |
| Et tertiæ triens | 1202 | |
| Et quartæ quadrans | 64 | |
| Et quintæ pars quinta | 4 | |

ſumma ———————0,0738374 eſt menſura rationis 8 ad 7,43058 16, eadem ſcilicet cum ratione 80000000 ad 74305816. Porro Log-us abſoluti 80000000 facile acquiritur ex ſuperiori tabella; cùm enim index primæ figuræ numeri 80000000 ſit 7, è regione 7rii ex ſecunda columna excerpo Log-um abſoluti 10000000 (hoc eſt unitatis ſeptem cyphris affectæ).

| | | |
|---|---|---|
| qui reperitur | 16, 11809565 | addo |
| cui ſubſcribo Log-um 8tii | 2,07944154 | |
| ſumma eſt Log-us abſoluti 80000000 | = 18, 19753719 | |
| ablata menſura rationis 80000000 ad 743005816 | = 0, 0738374 | |
| reſtat Log-us abſoluti 74305816 | = 18, 1236997, atque | |

tanta eſt area B I H F.

Mantiſſæ loco accipe modum facillimum quadrandi quamvis hyperbolæ partem per Log-os tabulares. Dati numeri 74305816 Log-us tabularis eſt 7,87102278, per ſuperioris tabellæ columnam ſecundam reducendus ad naturalem, proditque eadem, quæ ſupra, area B I H F = 18, 123699872.

Poſtremo, ne quis hæſitationi locus reſtet, accipe, quo pacto ex Prop. 13, 15, 16. Logarithmot. calculum ſuperiorem derivem.

Differentia terminorum rationem quamvis exprimentium ſi concipiatur diviſa in partes æquales innumeras; compoſita erit ratio tota extremorum terminorum ex innumeris ratiunculis terminorum à minimo ad maximum infinitiſſima parte ipſius differentiæ ſe mutuo excedentium. Sin iidem illi termini innumeri accipiantur pro mediis Arithmeticis aliorum terminorum ſimili parte infinitiſſima diſtantium; ſumma omnium ratiuncularum poſterioribus hiſce terminis intercedentium deficiet à tota ratione extremorum, non niſi ſemiſſe primæ & ultimæ ratiuncularum à prioribus terminis contentarum, id eſt, ratiuncula minori, quam quæ ullis numeris exprimi poſſit. Quare poſito Maximo termino = 1, & parte infinitiſſima differentiæ = i, & menſura rationis minimæ itidem i; erit ut medium Arithmeticum terminorum rationis minimam proxime præcedentis, ad medium Arithmeticum terminorum ipſius minimæ; ita menſura minimæ, ad menſuram proxime majoris; hoc eſt:

$1 - i \,.\, 1 :: i \,.\, i + ii + i^3 + i^4$ &c. menſuræ ultimæ
$1 - 2i \,.\, 1 :: i \,.\, i + 2ii + 4i^3 + 8i^4$ &c. penultimæ  add.
$1 - 3i \,.\, 1 :: i \,.\, i + 3ii + 9i^3 + 27i^4$ &c. antepenultimæ

fit ſumma ratiuncul. $= 3i + 6ii + 14i^3 + 36i^4$ &c. = numero terminorum,

rum, plus ſumma eorundem terminorum, plus ſumma quadratorum ab iiſdem, &c.

Sin minimus terminus ponatur $=$ 1, manentibus cæteris ut ſupra; evadit ſumma ratiuncularum $= 3i - 6ii + 14i^3 - 36i^4$, &c.

Hinc data differentia terminorum $= 0|\underline{1}$, erit numerus terminorum $= 0|\underline{1}$, & per 16 Logarithmot. ſumma eorundem terminorum $=$ 0, 005, & ſumma quadratorum $=$ 0, 000333. At data differentia terminorum $= 0|\underline{10}$; numerus terminorum eſt $=$ 0,01, & ſumma eorundem $=$ 0,00005, & ſumma quadratorum $=$ 0,00000333, &c.

*Nota*. Prop. IV. Logarithmot. Signa ſpeciebus intercedentia debebant eſſe alternatim affirmata & negata: atque ubicunque, Lector offenderit *infinitiſſimam*, legat *infiniteſimam*.

## Errata.

Page 742. *l.* 25. put a *comma* after *open'd*, (which is material for the ſenſe.) p. 749. *l.* 16. r. *idque*. ibid. *l* 40. r. *magnitudinum*. p. 753. *l.* 20. r. $-a + a^2, -a^3$, p. 754. *l.* 19. r. *Huic*. p. 755. *l.* 11. r. $b^2a^2 + b^2a^3 + b^2a^4$. ibid. *l.* 14. r $+ a^2 + a^3$. p. 756. in Fig. 1. the letters appearing obſcure, thoſe, that denote the ſmall lines parallel to the Aſymptote N A, are I B. *ps. qt. rn*. And the other capital letters are G F H. G B A. G M N.

In the *SAVOY*,

Printed by *T. N.* for *John Martyn*, Printer to the *Royal Society*, and are to be ſold at the *Bell* a little without *Temple-Bar*, 1668.